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The critical temperature of the two-dimensional $\pm J$ Ising spin glass

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Abstract

We have studied the two-dimensional $\pm J$ Ising model on the square lattice with a view to determine whether a spin glass can exist at small finite temperatures. By mapping the Ising model onto an ensemble of non-interacting lattice fermions, we have shown that the critical temperature for the spin glass phase transition appears to vanish. This result applies for all concentrations of negative bonds where the ground state is a spin glass.

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1. Introduction

One of the most widely studied examples of a spin glass is the short range $\pm J$ Ising model. The Hamiltonian is of the usual Edwards–Anderson [1] form

$$H = -\sum_{\langle ij \rangle} J_{ij}\sigma_i\sigma_j \tag{1}$$

where the nearest neighbour exchange interactions J_{ij} are quenched random variables of fixed magnitude but random sign distributed according to

$$P(J_{ij}) = p\delta(J_{ij} + J) + (1 - p)\delta(J_{ij} - J) \tag{2}$$

with the defect concentration $p \in [0, 0.5]$. For $p = 0.5$ on the square lattice it is well accepted that this model does exhibit spin glass behaviour at zero temperature. Furthermore, there are some indications [2, 3] that a spin glass occurs in the ground state for $p \neq 0.5$ also. The transition from ferromagnetism to the spin glass occurs at about $p_c = 0.11$ [3–12]. Spin correlations for the spin glass are expected to decay algebraically according to

$$[\langle S_0 S_R \rangle^2]_{av} \sim R^{-\eta} \tag{3}$$

with an exponent η which is most widely believed to take a value of about 0.2 at $p = 0.5$ [13–16]. Other work [3] predicts a constant value of η throughout the spin glass regime,

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although the estimate 0.34 is not in agreement. Estimates of η at the critical defect concentration p_c are about 0.18 [10, 12].

The existence of a spin glass at small finite temperatures is controversial. One body of work [17–20], using a cluster heat bath method for Monte Carlo simulations, has estimated that the spin glass transition occurs at a finite temperature $T_c \sim 0.23J$. Nevertheless, it is more widely accepted that the critical temperature should be zero [21–24]. In this paper, we present evidence that the critical temperature is zero independent of defect concentration p if the ground state is a spin glass.

2. Background

Analytical studies of the planar Ising model are well suited to one of the equivalent schemes for mapping onto an ensemble of non-interacting lattice fermions. The Pfaffian method [25] is one of these which is easily adaptable to the presence of disorder. It also has an advantage over networks [12] in that the zero temperature limit is more easily accessible. The Pfaffian method involves decorating the lattice with a set of four fermions, or Grassmann variables [26, 27], on each site and expressing the partition function as

$$Z = 2^N \left[\prod_{\langle ij \rangle} \cosh(J_{ij}/kT) \right] (\det D)^{1/2}. \tag{4}$$

The product is over all bonds on the N site lattice and D is a real, skew-symmetric matrix. The square root of the determinant of D is precisely the Pfaffian [25]. A real unitary transformation can be applied to cast D in a 2×2 block diagonal form where

$$D|\alpha\rangle = -\epsilon|\beta\rangle \qquad D|\beta\rangle = \epsilon|\alpha\rangle. \tag{5}$$

The ϵ are positive and referred to as eigenvalues of which there are $2N$.

In the zero temperature limit of the $\pm J$ model, it is only a subset of the eigenvalues which determine the changes in energy and entropy due to frustration [28]. These defect eigenvalues take the form

$$\epsilon = \tfrac{1}{2} X \exp(-2rJ/kT) \tag{6}$$

where X is a real number and r is an integer, both of which can be determined by degenerate state perturbation theory [28]. The number of defect states is exactly equal to the number of frustrated plaquettes [29]. The pairs of eigenstates corresponding to the defect eigenvalues are not necessarily unique due to arbitrary rotation of the basis which does not change the eigenvalue. Nevertheless, an invariant measure of spatial extent can be defined [3]. This is essentially the Manhattan distance between the centres of mass of the pair $|\alpha\rangle, |\beta\rangle$ with a correction to ensure invariance if overlap occurs.

Spatial extent is important since it is the emergence of extended states which destroys ferromagnetism and signals the transition to a spin glass. Specifically, it was found [3, 28] that the distribution of spatial extent, defined as the number of states with spatial extent greater than l, follows the power law

$$N(l) \sim L^2 l^{-\rho} \tag{7}$$

for a lattice of linear dimension L. If the fractal dimension $\rho < 2$ extended states are present. This delocalization of the lattice fermions indicates the spin glass state. To understand this, it is useful to express the spin correlation function in terms of reciprocal defects [30] along a path between two spins. Formally, we have

$$\langle S_0 S_R \rangle^2 = \det(1 - GV) \tag{8}$$

where $G = -D^{-1}$ is the Green function and V only has elements between fermions across a bond on the path. It is easy to appreciate that G will include long-range interactions in the presence of extended eigenstates. This of course gives a long-range influence to the disorder. It is also possible in principle to identify domains of disorder which can be solved independently [31]. Such domains will of course become spatially extended as a direct consequence of the emergence of extended eigenstates. It should also be understood that both dimensions are treated on an equal footing by the Pfaffian method which is only a very distant cousin of the transfer matrix formalism. Other applications of this type of delocalization have been discussed in relation to disordered superconductors and quantum Hall states [32, 33] as well as particle–hole localization [34].

3. Results

In this study, we have taken lattices with linear dimension $L \leqslant 32$ and diagonalized the matrix D directly. Bonds with infinite positive J were put on the boundary. This ensures that the boundary cannot be frustrated, while avoiding the errors associated with winding the lattice on a torus [25]. Essentially, the sample is embedded in an infinite unfrustrated lattice.

Data were collected for three sample sizes L (8, 16 and 32) and several defect concentrations p (0.11, 0.115, 0.12, 0.15, 0.2 and 0.5). Configurational averages over more than 500 random samples were taken for $L < 32$ and more than 50 for $L = 32$. Special care was taken to track the incidence of degeneracy in the eigenvalues which could result in spuriously extended states. Nevertheless, this problem was not found to be statistically significant.

The matrix D can be written exactly as $D(0) + \delta D_1$ where

$$\delta = 1 - \tanh(J/kT) \tag{9}$$

so that, if $kT \ll J$,

$$\delta \sim \exp(-2J/kT) \tag{10}$$

and $D(0)$ is D at zero temperature [31]. Figure 1 shows the distribution function $N(l)$ for $p = 0.2$ and $\delta = 0.01$ for three sample sizes L (8, 16 and 32) with data normalized to the $L = 32$ sample size. The data are not as well conditioned as for the ground state [3] due to the smaller sample size, although we believe that they do indicate the same behaviour. The fractal dimension ρ takes the value 1.66 ± 0.02 in agreement with the value found for the ground state [3]. Nevertheless, this scaling behaviour must be expected to break down at higher temperatures. Figure 2 shows what happens for $\delta = 0.02$. The data for $L = 32$ no longer scale and a discontinuity has appeared in the distribution function where $l \sim L$. It is clear that there are many more extended states than for $\delta = 0.01$. We believe that this behaviour indicates a breakdown of spin glass order since the correlation length ξ which limits scaling behaviour is now of the order of $L = 16$. The delicate balance between localization and extended states that exists in the spin glass phase has disappeared for $L > \xi$. Figure 3 shows the distribution functions at a higher temperature with $\delta = 0.05$. Only the data for $L = 8$ indicate the critical δ higher than 0.05 for this sample size. Figure 4 shows that all three distribution functions become discontinuous at $\delta = 0.1$. All standard error bars are smaller than the tokens in the figures.

For other defect concentrations $p > p_c$ the behaviour is much the same as at $p = 0.2$. The value of p_c, where the power law (7) breaks down, is estimated to be the same as in the ground state [3], with no evidence of temperature dependence. This is as expected from current knowledge of the phase diagram below the Nishimori point where $\delta \lesssim 0.2$ [11, 12].

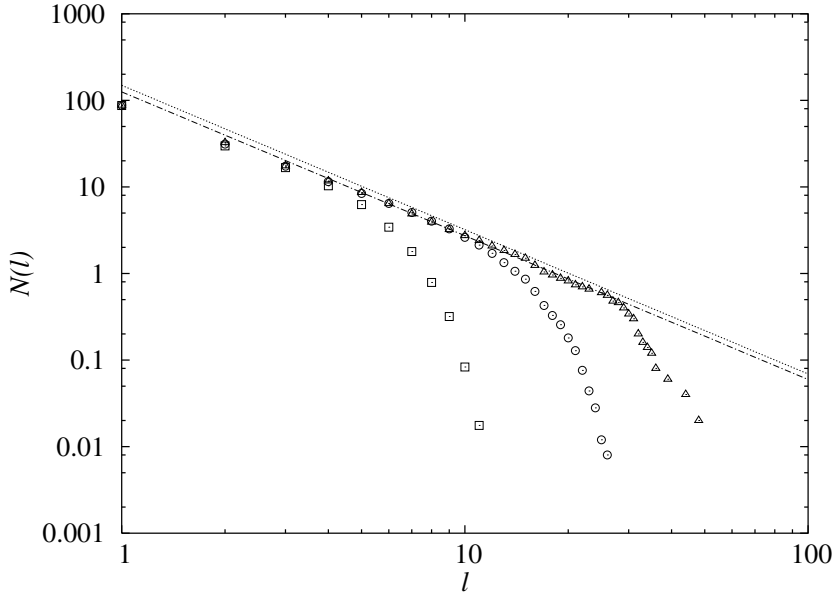

Figure 1. Distribution $N(l)$ (normalized to an $L = 32$ sample size) for $p = 0.2$ and $\delta = 1 - \tanh(J/kT) = 0.01$ for samples with $L = 8$ (squares), 16 (circles), 32 (triangles). Straight line fits to the $L = 32$ data for $p = 0.2$ (dotted line) and $p = 0.5$ (dot-dash line) are also shown with fractal dimension $\rho = 1.66$.

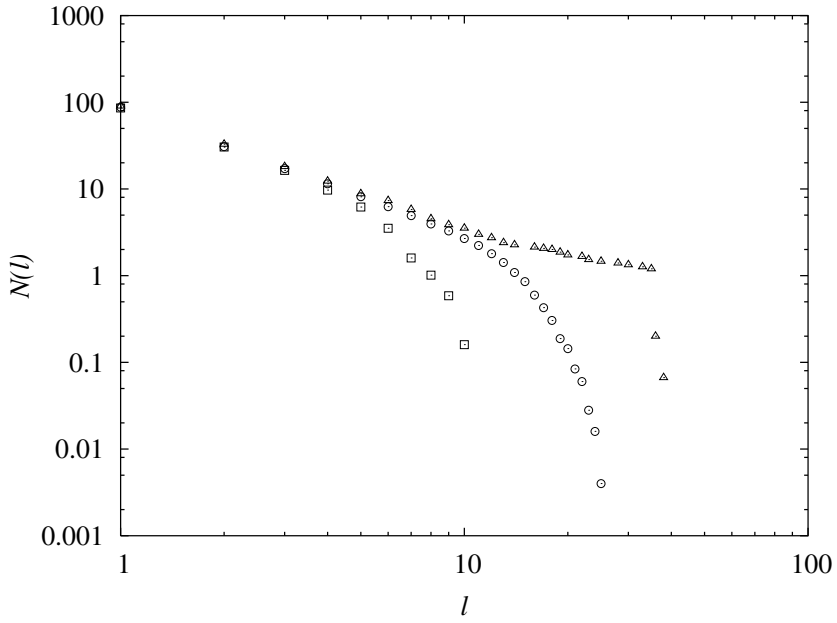

Figure 2. Distribution $N(l)$ (normalized to an $L = 32$ sample size) for $p = 0.2$ and $\delta = 0.02$ for samples with $L = 8$ (squares), 16 (circles), 32 (triangles).

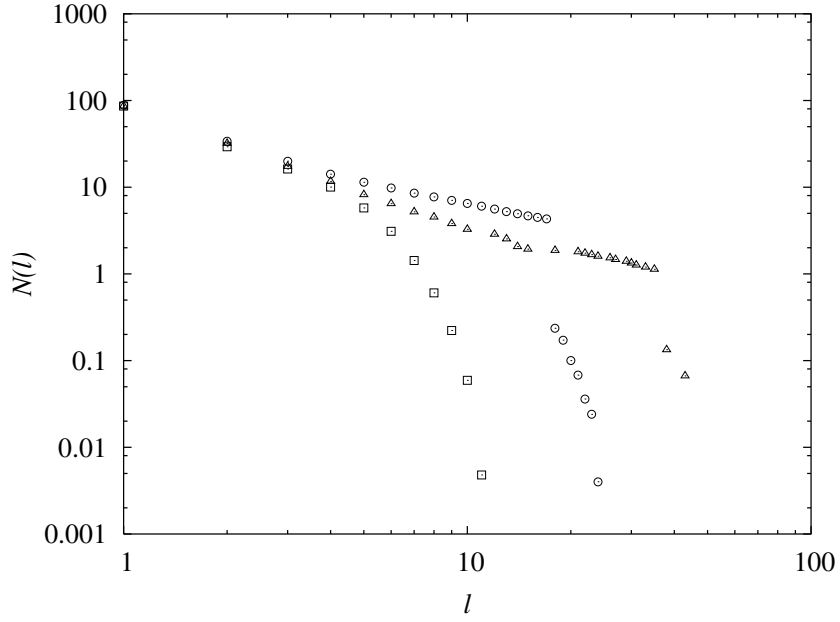

Figure 3. As figure 2 for $\delta = 0.05$.

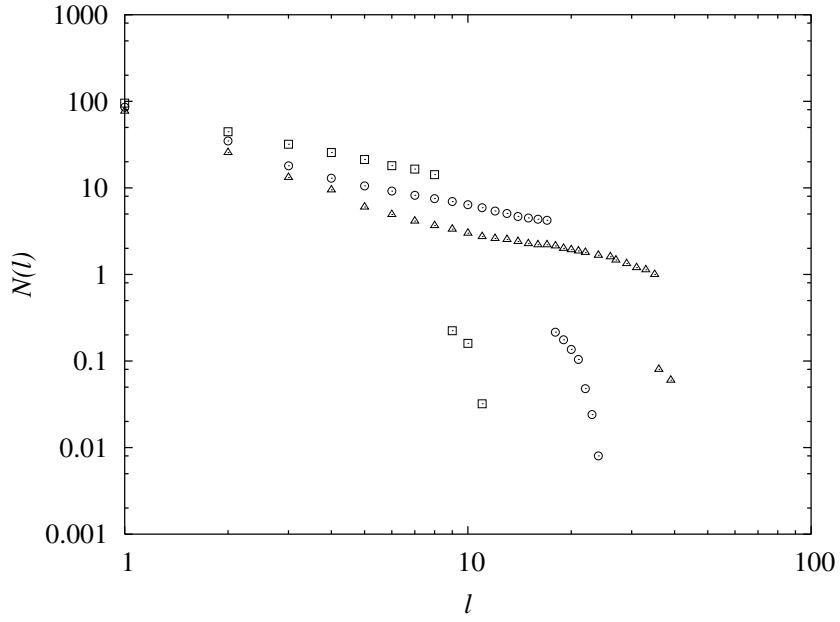

Figure 4. As figure 2 for $\delta = 0.1$.

We have searched for the critical value δ_c where scaling breaks down to within an error bar of 0.001. It is found that δ_c depends only on the sample size L, not the defect

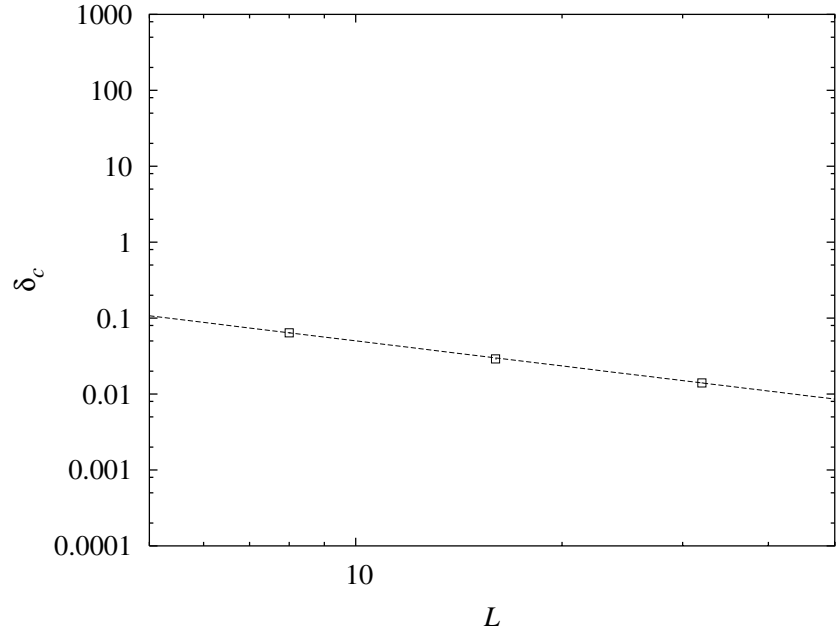

Figure 5. Plot of δ_c against L for $p = 0.12$.

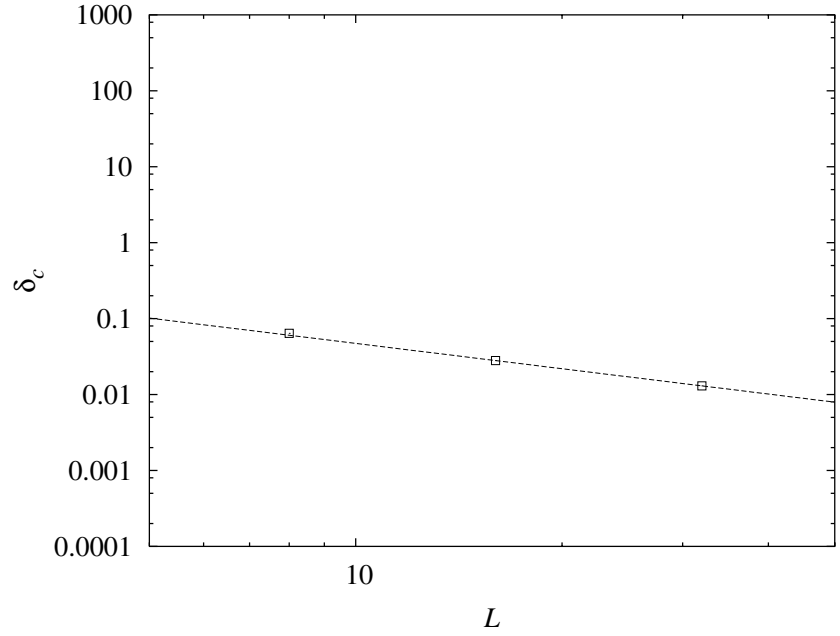

Figure 6. Plot of δ_c against L for $p = 0.5$.

concentration p. Figures 5 and 6 show plots of δ_c against L for $p = 0.12$ and $p = 0.5$. It is clear that we have a power law

$$\delta_c \sim L^{-\alpha} \tag{11}$$

where α is positive and independent of p. Line fits reveal a value $\alpha = 1.10 \pm 0.05$. Finite size scaling now indicates that $\delta_c = 0$ in the limit of infinite L. Hence the critical temperature of the spin glass phase appears to be zero, independently of p for $p > p_c$. In comparison, the cluster Monte Carlo method predicts $\delta_c = 0.0003$ [19] and $\delta_c = 0$ [24] for $p = 0.5$.

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